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VIBRATIONAL DISPLACEMENT IN A GRAVITY FIELD

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Vertical vibrational motion in a resistant medium in a gravity field is considered. Critical parameters for the motion regime in which the center of gravity of the system does not change its position with respect to the Earth surface are found.

Key words: *gravity field, vibrational motion, resistant medium.*

Introduction. Horizontal vibrational motion in a liquid was examined in [1, 2]; yet, the most interesting and general case of such a motion, the motion in a gravity field, remained unaddressed [3, 4]. So far, vibrational motion with resistance quadratic in velocity has been treated [3]. For very high oscillating frequencies of the unbalanced working body, the platform velocity relative to the medium can be substantial, which implies that a quadratic dependence of resistance on velocity. The reason why the model of [3] turned out to be incorrect consists in the physical meaning of the internal force that gives rise to vibrational motion. Nagaev and Tamm [3] put forward an explosion mechanism for excitation of unbalanced body oscillations; this mechanism involves a force, which is a sum of delta-functions shifted with respect to each other by the oscillation period. The model of [3] ignores recoil effects; hence, such a force cannot act as a periodic internal force. The integral of any periodic internal force over the oscillation period should be zero. The opposite statement would contradict not only the condition of periodic motion but also the law of conservation of momentum. Anyway, the study of vibrational motion in a gravity field due to a harmonic internal force seems to be of obvious interest.

Vibrational Motion in a Gravity Field. The presence of a medium resistant to motion in the direct and reverse directions is a necessary yet insufficient condition for vibrational displacement [4]. We assume that the resistance coefficient λ, which relates the resistance force $\boldsymbol{F}_r$ with the platform velocity $\boldsymbol{v}$,

$$\boldsymbol{F}_r = -\lambda \boldsymbol{v},$$

has different values for different directions of platform motion:

$$\lambda = \begin{cases} \lambda_+, & v > 0, \\ \lambda_-, & v < 0. \end{cases}$$

In addition to the resistance force, the platform of mass M experiences the action of the gravity force $M\boldsymbol{g}$ and the force $\boldsymbol{F}_{m-M}$ exerted by the unbalanced working body of mass m (see Fig. 1). The equation of motion for the platform is

$$M\,\frac{d^2\boldsymbol{y}}{dt^2} = M\boldsymbol{g} + \boldsymbol{F}_{m-M} + \boldsymbol{F}_r. \tag{1}$$

The working body experiences only the action of the gravity force $m\boldsymbol{g}$ and by the force $\boldsymbol{F}_{m-M}$ due to the platform:

$$m\,\frac{d^2(\boldsymbol{y} + \boldsymbol{y}_m)}{dt^2} = m\boldsymbol{g} + \boldsymbol{F}_{M-m} \tag{2}$$

($\boldsymbol{y}_m$ is the vector determining the position of the working body with respect to the platform). The position of the platform with respect to the Earth surface is given by the vector $\boldsymbol{y}$. Combining the equations of motion (1)

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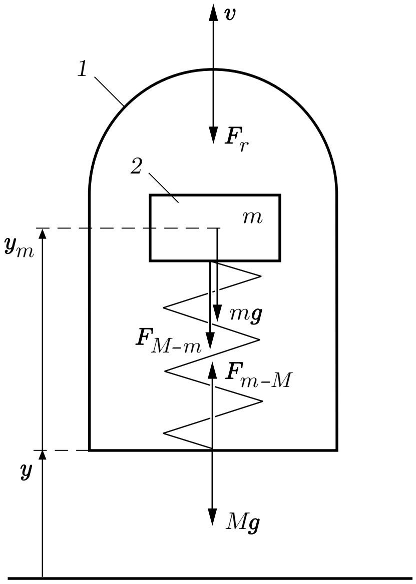

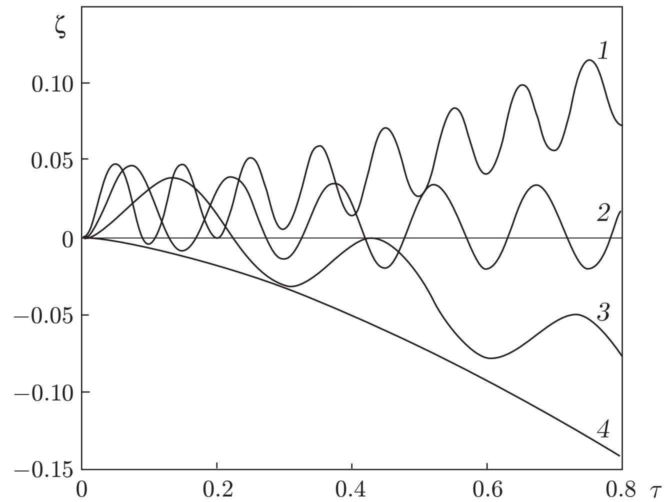

Fig. 1

Fig. 2

Fig. 1. Vibrational motion in a gravity field: 1) platform; 2) unbalanced working body.

Fig. 2. Platform coordinate ζ versus time for various oscillation periods of the working body: $\theta = 0.1$ (1), 0.152 (2), and 0.3 (3), and $\theta \to \infty$ (4).

and (2) and taking into account the relation $\boldsymbol{F}_{M-m} + \boldsymbol{F}_{m-M} = 0$, for the case of forced harmonic oscillations $[y_m = y_m^0 + a\cos(2\pi t/T)]$ executed by a body of mass m, the equation of motion for the platform can be written as

$$\frac{d^2\zeta}{d\tau^2} + \Big(\frac{1-\delta}{2}\,\mathrm{sign}\,\Big(\frac{d\zeta}{d\tau}\Big) + \frac{1+\delta}{2}\Big)\frac{d\zeta}{d\tau} + \eta - \frac{1}{\theta^2}\cos\frac{2\pi\tau}{\theta} = 0. \qquad (3)$$

Here η is the normalized gravity field: $\eta = M_0^3 g/(4\pi^2 ma\lambda_+^2)$ and

$$\zeta = \frac{M_0}{4\pi^2 ma}\,y, \quad \delta = \frac{\lambda_-}{\lambda_+}, \quad \tau = \frac{\lambda_+}{M_0}\,t, \quad \theta = \frac{\lambda_+}{M_0}\,T, \quad M_0 = m + M.$$

In spite of the nonanalytical nature of the coefficient at the normalized velocity $d\zeta/d\tau$ in (3), the numerical solution of this equation presents no difficulties. The solution of Eq. (3) for $\delta = 4$ and $\eta = 1$ is exemplified in Fig. 2, which shows three regimes of vibrational motion in a gravity field. For high oscillation frequencies of the working body, the system can ascend over the surface (curve 1). Low frequencies correspond tp the downward motion of the system in the gravity field (curve 3). The descent here is not free because of the resistance of the medium and the influence of the oscillating working body. For comparison, Fig. 2 shows the normalized coordinate ζ versus time for zero oscillating frequency, i.e., for the case in which the working body does not vibrate (curve 4). There exists a critical regime of vibrational motion, in which the center of mass remains stationary over the Earth surface (curve 2). The main objective of the present work is to give a description of this regime.

Critical Regime of Vibrational Motion. The critical regime separates two regimes of vibrational motion. The ascent of the system in the gravity field is characterized by a positive value of mean velocity $\langle v\rangle$ in vibrational motion of this type. Conversely, negative values of mean velocity $\langle v\rangle$ correspond to downward motion of the platform. Hence, to establish the conditions for the critical regime of vibrational motion in the gravity field, it suffices to find the conditions under which the mean velocity is zero. The following specific feature in calculating this parameter is worth noting: if the value of η is high, then the mean normalized velocity $\langle\vartheta\rangle = M_0^2\langle v\rangle/(4\pi^2 ma\lambda_+)$ vanishes only at short dimensionless oscillation periods θ. In turn, the latter means that setting a fixed motion-stabilization time interval after which the mean (averaged over a period) velocity of vibrational motion is calculated [1] is inadmissible. The time required for the motion to become stable should be considered as a calculation parameter to be found from the condition that the mean velocity is independent of the value of this parameter.

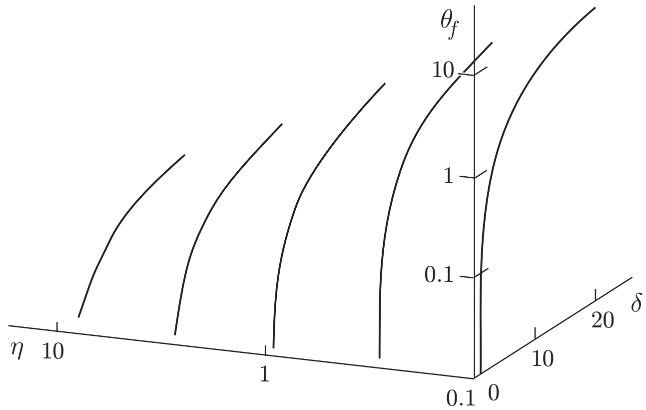

Fig. 3. Critical dimensionless oscillation period θ_f versus the parameter of asymmetry δ.

Figure 3 shows the dimensionless oscillation period θ_f in the critical regime as a function of η and δ. Actually, these data were obtained by numerically solving the equation

$$\langle\vartheta\rangle(\eta, \delta, \theta_f) = 0 \tag{4}$$

for various values of η and δ. To differentiate between the values of the dimensionless period in the critical regime and the values of the parameter θ for upward and downward motions of the system, we introduced a new parameter θ_f. As one would expect, if the total mass of the system is small, then the critical regime of vibrational motion starts already at $\delta \approx 1$. If the resistance coefficient λ_+ for the upward motion of the platform is much lower than the resistance coefficient λ_- for its downward motion, then the critical regime of vibrational displacement can occur at rather low oscillating frequencies of the unbalanced working body. Finally, the critical regime of vibrational motion of a heavy system (high values of η) emerges only at sufficiently high oscillating frequencies or, in other words, at low θ_f.

It should be noted that the data of Fig. 3 are of low practical significance. A convenient way to represent experimental and theoretical results is provided by the self-similar approach [1, 2] based on the symmetry of the horizontal vibrational motion with simultaneous substitution of λ_+ and λ_-, respectively, for λ_- and λ_+. If the system moves vertically (i.e., in a gravity field), then this symmetry is violated. The rule of transformation remains unchanged only for the dimensionless period θ_f. This rule implies that the universal variable for the critical vibration period has the form $\theta_{\delta f} = \theta_f/(1+1/\delta)$. To write the corresponding variable for the normalized gravity force of the whole system η, it is necessary to take into account that, following the simultaneous replacement $\lambda_+ \leftrightarrow \lambda_-$, the equation of motion (3) retains its form only if the quantity η also changes its sign. At the same time, the value of η quadratically depends on the resistance coefficient. This means that the transformation of the squared resistance coefficient, λ_+, rather than that of the resistance coefficient λ_+^2 itself, should be antisymmetric. The antisymmetric transformation of velocity of vibrational motion in the horizontal direction corresponds to the replacement $\lambda_+ \to \lambda_+\lambda_-(\lambda_- - \lambda_+)/(\lambda_- + \lambda_+)^2$ [2]. The transformation of the normalized gravity field should correspond to an analogous replacement, since the critical regime of vibrational motion in the first-order approximation can be represented as superposition of two motions: the downward motion of the system in the gravity field with zero oscillation frequency and vibrational counter-displacement with $\eta = 0$. The only replacement that suits this model and preserves the dimensions of the resistance coefficient of the medium has the form

$$\lambda_+^2 \to \lambda_+\lambda_-(\lambda_- - \lambda_+)/(\lambda_- + \lambda_+).$$

Then, the normalized gravity force can be written as the variable

$$\eta_\delta = \eta(\delta + 1)/(\delta(\delta - 1)),$$

which depends, as it could be expected, only on the dimensionless oscillation period $\theta_{\delta f} = \theta_f/(1 + 1/\delta)$ in the critical regime of vibrational motion.

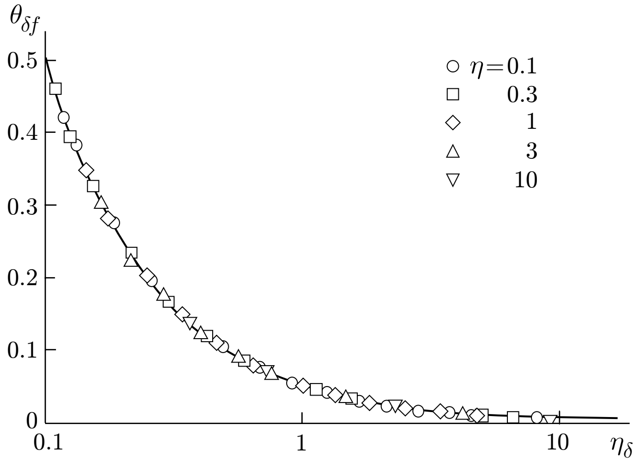

Fig. 4. Self-similar dependence of the critical oscillation period $\theta_{\delta f}(\eta_\delta)$: the points refer to the solution of Eq. (4) for $1.1 \leqslant \delta \leqslant 10$ and $0.1 \leqslant \eta \leqslant 10$; the curve is calculated by formula (5).

The above-mentioned approach can be used to approximately describe the calculation data plotted in Fig. 4 as the dependence $\theta_{\delta f}(\eta_\delta)$. Here, two important circumstances are worth noting. First, for the normalized gravity force changed by more than two orders of magnitude and for the asymmetry parameter of the system changed more than tenfold, the critical dimensionless oscillation period is given by the simple relation

$$\theta_{\delta f} \approx 1/(20\eta_\delta), \tag{5}$$

which can be written as

$$T_f \approx \pi^2 ma(\lambda_- - \lambda_+)/(5M_0^2 g).$$

Second, the above-described approach, based on the property of symmetry, proved to be valid not only for long dimensionless periods and high parameters of asymmetry. The latter seems to be caused by the general behavior of dependence (5), which, actually, leads to the fact that the self-similar representation is not unique.

If, in the absence of any oscillations of the unbalanced body, the maximum velocity of the downward motion of the whole system in air with the resistance coefficient λ_- is v_m, then we have $\lambda_- v_m = M_0 g$. This means that vibrational ascent of such a system in the gravity field can be obtained for oscillation periods shorter than $T_f = \pi^2 ma(1 - 1/\delta)/(5M_0 v_m)$. For the mass ratio $m/M_0 = 1/2$, oscillation amplitude $a = 1$ m, parameter of asymmetry $\delta = 2$, and velocity $v_m = 1$ m/sec, we have $T_f \approx 0.5$ sec.

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